ECONOMICS RELATED TO EXPLOITATION OF GAS WELLS AT CONSTANT PRESSURE

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INTRODUCTION

The problem of gas flow through porous mediums with application to exploitation of fields shows serious difficulties even in the simple case of plane-quaquaversal flow through a homogenous and isotropic porous medium. Indeed, even if it accepted that the flowing phenomenon is usually isothermal; the pressure differential equation is non-linear and as for the gas behaviour both the deviation from the perfect gases low and the viscosity pressure variation should be taken into account.

The mathematical model described in this study takes into account all the above aspects; therefore the use of a numerical method of solving is required. Such method is applied in the case of a permeable field exploited at various rates of flow. For each exploitation pressure is thus established and its optimal value can be found.

MATHEMATICAL MODEL

The process of gas plane-quaquaversal isothermal flow through a homogenous and isotropic porous medium is simulated by equation [1]

$$\nabla\left(\frac{p}{\mu Z}\nabla p\right) = \frac{m}{k}\frac{\partial}{\partial t}\left(\frac{p}{Z}\right) \tag{1}$$

where ∇-Hamiltonian, m and k – porosity, i.e. the permeability of porous medium(constants), p – field gases pressure, and μ and Z – dynamic viscosity, i.e. the factor of gas deviation from the perfect gas model, both depending on pressure. Equation (1) may also be written as

$$\nabla \cdot (\varphi \nabla p) = \psi \frac{\partial p}{\partial t} \tag{2}$$

where the function φ and ψ result from the relations

$$\varphi = \frac{p}{Z\mu}; \psi = \frac{m}{k}\frac{1}{z^2}\left(Z - p\frac{dZ}{dp}\right) \tag{3}$$

and depend on gas pressure only. We can also write

$$\nabla \cdot (\varphi \nabla p) = \frac{\partial \varphi}{\partial t}(\nabla p)^2 + \varphi \nabla^2 p$$

which results in making equation (2) becom

$$\frac{\partial \varphi}{\partial t}(\nabla p)^2 + \varphi \nabla^2 p = \psi \frac{\partial p}{\partial t} \tag{4}$$

As $p = p(r,t)$, by change into polar coordinates, equation (4) becomes

$$\frac{\partial^2 p}{\partial p^2} + \frac{1}{r}\frac{\partial p}{\partial r} - \frac{\psi}{\varphi}\frac{\partial p}{\partial t} = -\frac{1}{\varphi}\frac{d\varphi}{dp}\left(\frac{\partial p}{\partial r}\right)^2 \tag{5}$$

Equation (5) represents the mathematical model of the process of gas flow through a circular field towards a central well, its solution providing the gas pressure distribution in time according to radius. Introducing a new dimensionless variable by

$$\xi = \frac{\ln \bar{r}}{\ln \overline{R_c}}\,;\ \bar{r} = \frac{r}{R_s}\,;\overline{R_c} = \frac{R_c}{R_s} \tag{6}$$

R_s being the well radius and R_c the field outline, as well as the low pressure $P(\xi,t)$ by

$$P(\xi,t) = \frac{p(\xi,t)}{p_c} \tag{7}$$

p_c being the critical pressure, equation (2) becomes [2]

$$\frac{\partial^2 P}{\partial \xi^2} - a\left(\overline{R_c}\right)^{2\xi} \Phi \frac{\partial P}{\partial t} = -b\left(\frac{\partial P}{\partial \xi}\right)^2 \Psi \tag{8}$$

where the functions Φ and Ψ result from the expressions

$$\Phi(P) = \mu\left(\frac{1}{P} - \frac{1}{Z}\frac{\mathrm{d}Z}{\mathrm{d}P}\right);\Psi(P) = \left(\frac{1}{P} - \frac{1}{Z}\frac{\mathrm{d}Z}{\mathrm{d}P} - \frac{1}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}P}\right) \tag{9}$$

and the coefficients a and b are calculated with

$$a = \frac{m\mu_0}{kp_c} R_s^2 \ln^2\left(\overline{R_c}\right);\ b = \frac{1}{8p_c}. \tag{10}$$

Equation (8) simulating the process of gas flow towards a central well into a homogenous and isotropic circular field is a differential equation with partial derivative of 2^{nd} degree of parabolic type and non-linear. It admits no analytic solution, but a computer assisted numerical approach.

INITIAL AND LIMIT CONDITIONS

The initial condition transposes mathematically the fact that the gas pressure in entire field has the value p_z at the initial moment, i.e.

$$P(\xi,0) = P_z = \frac{P_z}{P_c} \tag{11}$$

The limit conditions are necessary in terms of field exploitation method and also of its characteristic. Hence, in case the field is exploited at constant rate of flow, the condition of maintaining constant the gas pressure p_s at well outlet is to be set:

$$P(0,t) = P_s = \frac{p_s}{p_c} \tag{12}$$

In the case of closed field with permeable outline(with water pushing), the condition of maintaining constant the pressure p_z on such outline is necessary,i.e.

$$P(1,t) = P_z = \frac{p_z}{p_c} \tag{13}$$

NUMERICAL METHOD

We will transform the continuous spectrum C: $[0 \le \xi \le 1,\ 0 \le t \le T]$ into the point discrete lattice R_{ij} :[$\xi=(i-1)\cdot h$, $t=j\cdot\tau_i n_i$], where $i=1\div n$, $j=0\div m$ are the spatial index and the temporal index, respectively, and h and τ are the spatial pitch and temporal pitch, respectively, and n and m are their numbers. Thus, instead of exact values of pressure $P(\xi, t)$ we will consider the discrete approximate values $P_i^j = P\left(\xi_i, t_j\right)$.

Making use of calculus scheme with finite quotients of Hyman Kaplan implicit type [3], known as stable unconditionally and absolutely convergent, as well as end approximations proposed by West, Garvin and Sheldon [2], equation (8) becomes a system of equations generated by the scheme, for $i = 2 \div n$, where

$$P_{i-1}^{j+1} - \left(2 + C_i^{j+1}\right) P_i^{j+1} + P_{i+1}^{j+1} = -C_i^{j+1} P_i^{j} - D_i^{j+1} \tag{14}$$

$$C_i^{j+1} = \frac{ah^2}{\tau}\left(\overline{R_c}\right)^{2h(i-1)} \Phi\left(P_i^{j+1}\right); D_i^{j+1} = \frac{b}{4}\Psi\left(P_i^{j+1}\right)\left(P_{i+1}^{j+1} - P_{i-1}^{j+1}\right)^2 \tag{15}$$

$$\Phi\left(P_i^{j+1}\right) = V_i^{j+1}\left(\frac{1}{P_i^{j+1}} - \frac{Zp_i^{j+1}}{Z_i^{j+1}}\right), \ \Psi\left(P_i^{j+1}\right) = \left(\frac{1}{P_i^{j+1}} - \frac{Zp_i^{j+1}}{Z_i^{j+1}} - \frac{Vp_i^{j+1}}{V_i^{j+1}}\right) \tag{16}$$

$$V_i^{j+1} = \mu\left(P_i^{j+1}\right); \ Vp_i^{j+1} = \frac{\mathrm{d}\mu}{\mathrm{d}P}\left(P_i^{j+1}\right), \tag{17}$$

$$Z_i^{j+1} = Z\left(P_i^{j+1}\right); \ Zp_i^{j+1} = \frac{\mathrm{d}Z}{\mathrm{d}P}\left(P_i^{j+1}\right). \tag{18}$$

completed with

$$P_i^{j} = Pz \,; P_1^{j+1} = P_s \,; \ P_{n+1}^{j+1} = Pz \tag{19}$$

Here the values of low pressure at temporal level j are considered as known, and unknown at temporal level $j+1$, respectively.

NUMERICAL SIMULATION OF FIELD EXPLOITATION

In order to solve the system of linear algebraic equations generated by the scheme with finite quotients (14) we will mark with $P0(i)$ the distribution of low pressure corresponding to known moment, j, and with $P(i)$ the distribution of low pressure corresponding to unknown moment, $j+1$. The two approximate solutions successively obtained by solving the algebraic system generated by the calculus scheme adopted will be marked with $P1(i)$ and $P2(i)$, respectively; obviously i will take values from 1 to $n+1$. To facilitate the compiling of the calculus program we will define the supporting functions $Z(X)$, $Zp(X)$, $V(X)$, $Vp(X)$ that will help us calculate the value sets of deviation factor $Z(i)$, $Zp(i)$ and dynamic viscosity $V(i)$, $Vp(i)$, respectively, corresponding to the distribution of low pressure at moments required by running the calculus program, i.e. for $P0(i)$, $P1(i)$or $P2(i)$. Now the values corresponding to expressions $C(i)$ and $D(i)$ can be calculated based on calculus procedures. The conditions (19) are obviously written as

$$P0(i) = Pz \,; \ P(1) = Ps; \ P(n+1) = Pz \tag{20}$$

In the case of permeable closed field exploited at constant pressure, the calculus scheme (14).
With the conditions (20) generate the following system of equations

$$P(1) = Ps$$

$$P(i-1) - \left[2 + C(i)\right] P(i) + P(i+1) = B(i) = -C(i)P0(i) - D(i), \ i = 2 \div n$$

$$P(n+1) = P_z \tag{21}$$

As the resulting algebraic system has a coefficient matrix of Jacobi type, i.e. tridiagonally, its solving becomes easy following the use of Thomas process [2]. Thus, we will firstly determine

$$a'(1) = 0 \,; \ b'(1) = P_s \tag{23}$$

after which the following can be calculated for $i = 2 \div n$

$$a(i) = -\left[2 + C(i)\right]; \ b(i) = -C(i)P0(i) - D(i) \tag{23}$$

$$a'(i) = \frac{1}{a(i) - a'(i-1)}; \ b'(i) = \frac{b(i) - b'(i-1)}{a(i) - a'(i-1)} \tag{24}$$

and finally, because $b(n+1) = P_z$, the solution of the system of equation is

$$P(n+1) = P_z;\ P(n-i+1) = b'(n-i+1) - a'(n-i+1)P(n-i+2) \qquad (25)$$

CALCULUS EXAMPLE

We will consider a closed impermeable circular gas field having the outline radius of 200 m, 40 m thickness, 20% porosity and 10 mD permeability. The pressure of field gases is of 140 bar, and their temperature of 27 °C. The field is exploited by a well of 0,1 m radius.

For pressure depending on gas viscosity and deviation factor, respectively, we accept the relations:

$$\mu(P) = 1 + 0{,}02P + 0{,}004P^2 \qquad (26)$$

$$Z(P) = 1 - 0{,}062P + 0{,}004P^2 \qquad (27)$$

experimentally determined from field curves. We will consider a spatial pitch of digitization lattice of 0,01 a temporal pitch of 60 s, and 10^{-5} acceptable error of iterative calculus.

In diagram 1 the gas cumulation for a pressure values set is showed. It can be observed the range of optimal pressures being included between 65 and 70 bar.

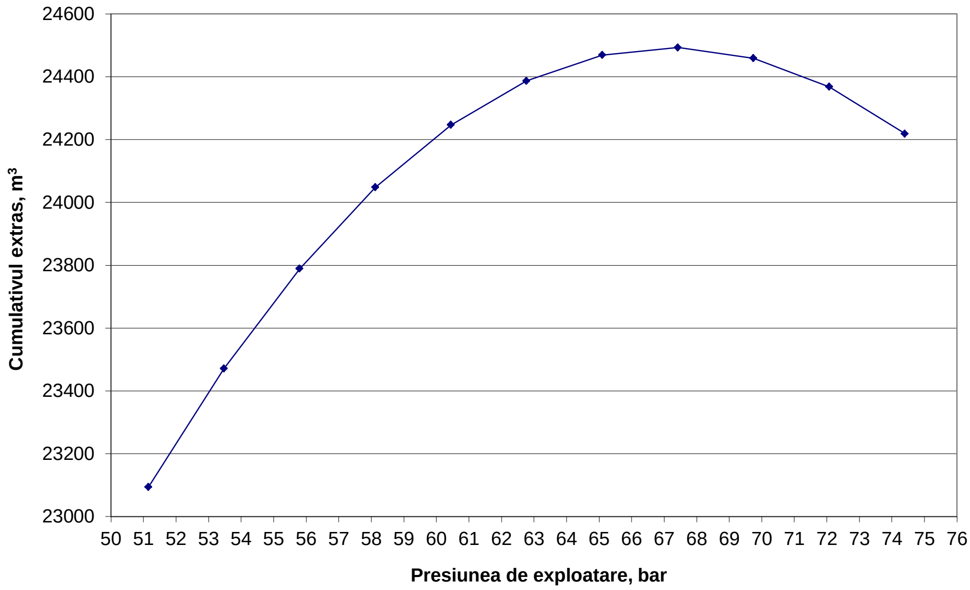

Diagram 1

CONCLUSIONS

The model proposed may be used to exploit gas fields through wells at constant pressures.

The calculus program may be included as equipment of extraction field, simulating the deposit exploitation at various gas pressures at top of the well, and based on daily cumulation the pressure leading to maximum output can be chosen.

ABSTRACT

It is regarded the isotherm movement of gases through a porous and isotropic medium towards a central well, taking into account the deviation from the perfect gases law and the viscosity pressure variation. The resulting model, completed with the specific limit conditions, is approached through a numerical method of solving and is applied to the wells through which the gas fields are exploited at constant pressure. For the current exploitation pressures daily cumulations of gases have been determined, thus resulting the optimal value of the exploitation pressure.

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SOME ASPECTS REGARDING THE STRESS DISTRIBUTION IN THE VESSEL ELEMENT – ASYMMETRIC CONIC REDUCTION PART II – FEM ANALYSIS

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The calculus of the asymmetric conical reductions that are part of the heat exchangers with steam space (Kettle type reboiler - see figure 1) that have to withstand interior pressure, is done using the C4 – 90 [1] standard. The thickness of the wall of the asymmetric conical reduction, during the design process is determined with the relation:

$$s \geq s_c + c_1 + c_t + c_a \qquad (1)$$

The significance of the symbols in equation (1) are: c_1 represents the corrosion thickness supplement; c_t – the negative deviation of the metal sheet thickness; c_a – technological supplement; s_c – the thickness of the metal sheet determined from the failing condition with the relation:

$$s = \frac{pD}{2\sigma_a Z - p} \cdot \frac{1}{\cos\alpha} \qquad (2)$$

In equation (2), D is the interior diameter of the conical element – see figure 2, *a*; p – the calculus pressure; Z – the failing coefficient of the welded hinges; α – the angle of the generatrix in the vertical symmetry plane (figure 2). The calculus admissible tension σ_α is determined with the relation:

$$\sigma_a = \min\left[\frac{R_e^t}{c_{s1}}; \frac{R_m^t}{c_{s2}}\right] \quad (3)$$

Where R_e^t is the apparent yielding limit of the sheet metal material at the working temperature t; R_m^t - tensile strength of the sheet metal material at temperature t; c_{s1} and c_{s2} – safety coefficients.

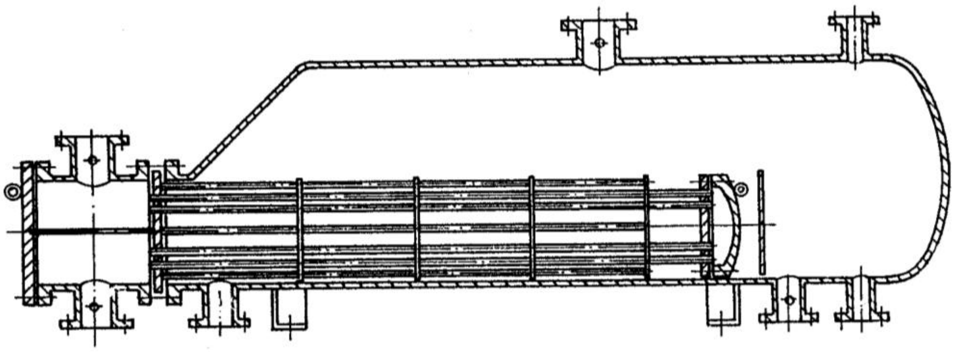


Figure 1

Equation (2) responds to the PD 5500:2001 [2] where the tensions that appear in the symmetrical conical reductions (figure 3) are determined with the relations:

$$\sigma_\theta = \frac{pD}{s} \cdot \frac{1}{\cos\alpha}; \quad (4)$$

$$\sigma_z = \left(\frac{pD^2}{2(D+s)s} \cdot \frac{1}{\cos\alpha}\right) + \left(\frac{W}{\pi(D+s)s} \cdot \frac{1}{\cos\alpha}\right) + \left(\frac{4M}{\pi(D+s)^2 s} \cdot \frac{1}{\cos\alpha} \cdot \cos\theta\right); \quad (5)$$

$$\tau = \left(\frac{4M}{\pi(D+s)^2 s} \cdot \tan\alpha \cdot \cos\theta\right) + \frac{2T}{\pi(D+s)^2 s}, \quad (6)$$

where W is the axial force on shell (positive if tensile) at the transverse section considered (this force excludes pressure load); M is the bending moment on shell acting in a plane containing the shell axis, at the transverse section considered; T is the torque acting about shell axis at transverse section; θ is the angle included by plane of action of moment M and an axial plane through the point considered.

As can be seen the calculus of the asymmetric conical reductions that hat are part of the heat exchangers with steam space (figure 1 and 2) is identical with the one used in the case of the symmetrical conical reduction. This calculus does not take into account, on one hand, the reduction asymmetry, and on the other hand, the tension amplifications that appear in the filleting zone.

For a correct evaluation of the tensions in such a structure one used the FEM method.

The problem of constructing the model, as emphasized in the first part of the paper [3] proved difficult, the solution being obtained by an interdisciplinary approach, using Solid Edge and Excel.

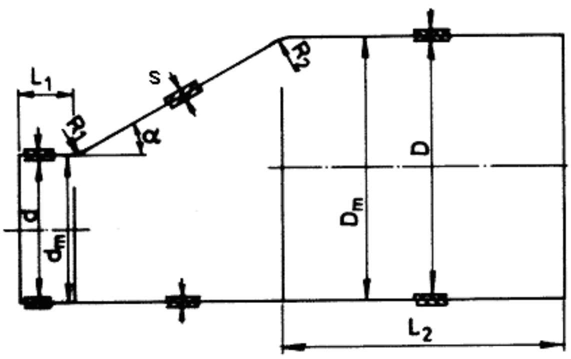


Figure 2

For a case study, one has chosen the heat exchanger with the following geometrical dimensions: d_i = 1400 mm; D_i = 2800 mm, α = 45°.

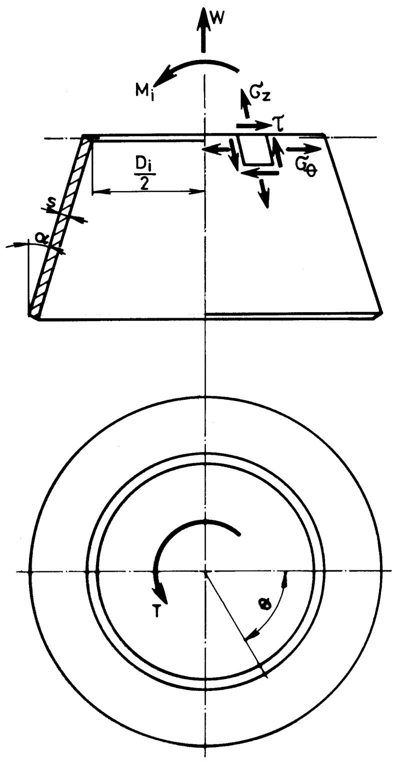
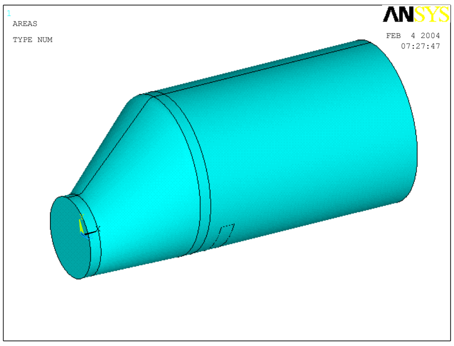


Figure 3

THE MODEL

The geometric model of the conical zone has been imported from Solid Edge in ANSYS 7.0. The model has been completely defined through the adding of the cylindrical zones directly related with the conical zone, along with a cylindrical zone corresponding to the heat exchanged body, sufficiently lengthy not to influence the tensions in the conical area and in the support area (figure 4).



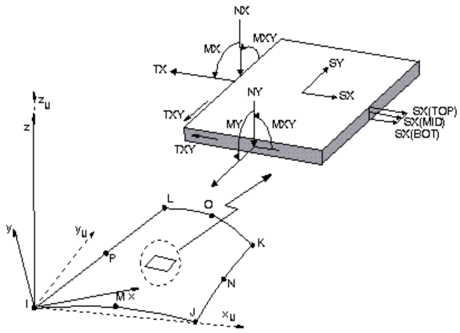


Figure 4 *Figure 5*

ELEMENTS

The authors considered the SHELL93 element (figure 5) from the elements library of ANSYS 7.0, an eight nodes element, that allows determining the tensions and deformations in the moment theory. For the considered element one has defined four thicknesses, corresponding to different zones of the model, thicknesses obtained by design calculus. So: for the cylindrical zone *s* = 25 mm; for the saddle support *s* = 45 mm; for the asymmetric conical zone *s* = 35 mm; for the flange *s* = 120 mm.

LOADS AND RESTRAINTS

One has defined two loading cases corresponding to the real loads of the heating exchanger:

a) interior testing pressure *p* = 3 MPa and temperature *t* = 20^{o}C; b) interior working pressure *p* = 1,93 MPa and temperature *t* = 177^{o}C.

As far as the displacement restraints are concerned, all the nodes corresponding to the saddle support were blocked, along with the rotations in a plane perpendicular to the cylindrical axis and (figure 6).

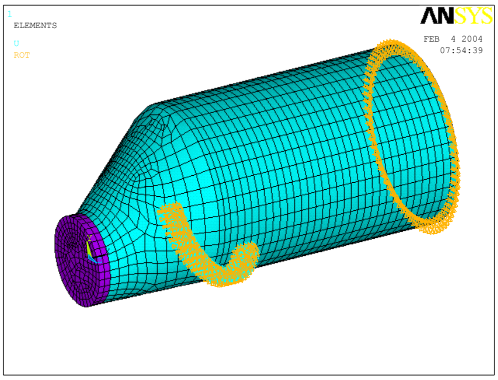


Figure 6

THE MATERIAL

The material used for manufacturing the conical reductions is P255GH (SR EN 10028). The characteristics are presented in table 1.

One has considered that the used material withstand a plastic deformation with nonlinear cold hammering and the characteristic deformation-tension defined by the equation (see also figure 7):

$$\sigma = \begin{cases} E \cdot \varepsilon & \text{pentru } \varepsilon \le \varepsilon_p = 0,002 + R_e / E \\ K \cdot \varepsilon^n & \text{pentru } \varepsilon > \varepsilon_p \end{cases}, \qquad (7)$$

where: *E* is the Young modulus, *n* – cold hammering exponent; *K* – strength coefficient.

Table 1

Temp. [^oC]	R_e [MPa]	R_m [MPa]	*E* [MPa]
20	255	410....490	205940
200	206	300...358	-
250	180	-	-
300	157	-	181420
350	137	-	-
400	118	-	171620
450	98	-	-
500	-	-	161810
600	-		152000

In the case of a nonlinear behavior of the material one can write:

$$K\left(0,002 + \frac{R_e}{E}\right)^n = R_e, \qquad (8)$$

$$K\left(\frac{A}{100}\right)^n = R_m, \qquad (9)$$

where *A* is the total strain.

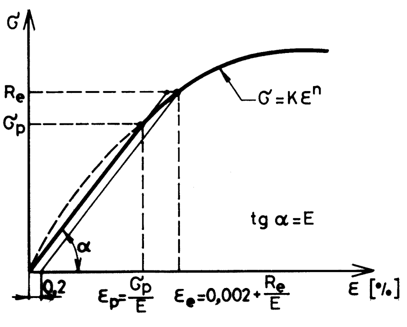


Figure 7

Equations (8) and (9) constitute a system that allows determining the unknowns *n* and *K*. For the material P255GH at the ambient temperature one has considered R_e = 255 MPa, R_m = 410 MPa and *A* = 22%. With these data, one obtained the following values for *n* and *K*, *n* = 0,1216; *K* = 341,05 MPa, values used to model, in ANSYS, the nonlinear behavior of the material. The same approach has been considered for the mechanical characteristics of the material at temperature $t = 177^{o}$C .

INTERPRETATION OF THE RESULTS

The analyzed results correspond to the two loading cases presented above. The monitored values were the maximal tensions σ_{INT} (Tresca) and σ_{EQV} (von Mises) in the vertical symmetry plane of the conical asymmetric reduction following the generatrix for which $\alpha = 45^{o}$. In figure 8 one presents the distribution of the tensions σ_{EQV} for case *b*) of loading.

In figure 9, one presents the variation of the principal tensions at the exterior, respectively interior of the conical asymmetric shell for the loading case *a*), and in figure 10 for case *b*).

In table 2, are presented the maximal principal tensions obtained analytically and by using FEM.

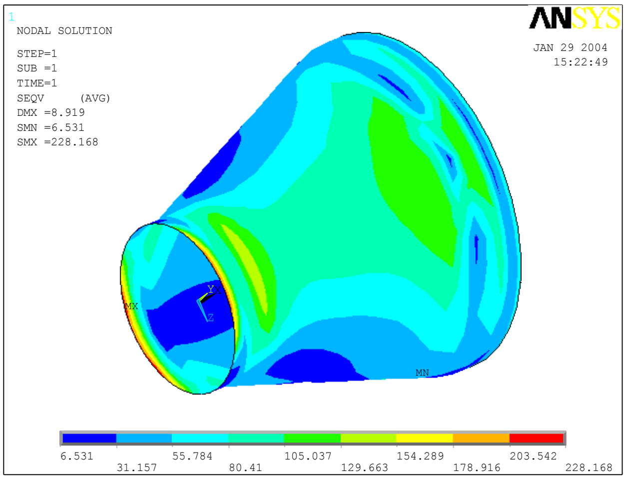


Figure 8

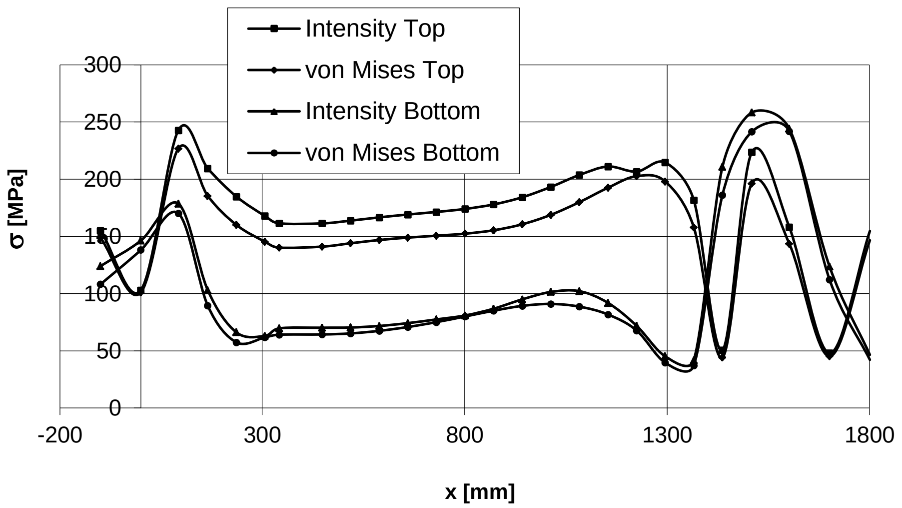


Figure 9

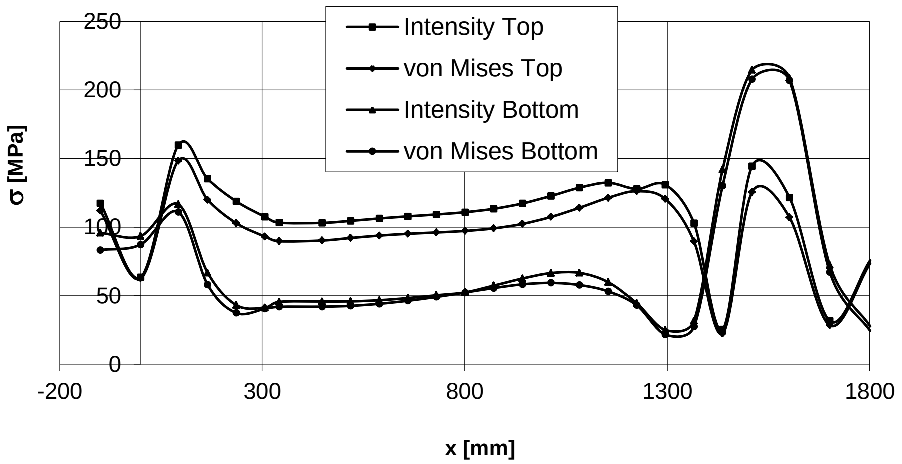


Figure 10

Table 2

	p = 3 MPa; *t* = 20°C		*p* = 3 MPa; *t* = 177°C	
	σ_{INT} [MPa]	σ_{EQV} [MPa]	σ_{INT} [MPa]	σ_{EQV} [MPa]
Analitic	121,5	105,22	114,5	99,6
FEM	248	241	214,5	207,8

CONCLUSIONS

Firs of all, one can mention the significant difference between the tensions obtained using design formula and FEM analysis. This situation leads to the following observations:

- the thickness *s* of the sheet metal used to construct the asymmetric conical reductions obtained through the analytical method leads to values of the principal tensions far smaller than the real ones;

- the values of the real tensions, as they were obtained through FEM analysis are still within acceptable limits if one considers the mechanical characteristics of the material used (see table 1) with the amendment of modifying the safety factors c_{s1} = 1,5 and c_{s2} = 2,5 to 1,03 respectively 1,7.

On the other hand, one has to stress the concentration of tensions in the filleting areas, where the round radiuses have the smallest values (for the case study r = 170 mm). The influence of the round radius r is insignificant, the tensions obtained on a model with r = 200 mm are very closet o those obtained on the initial model.

It is also important to mention the contour effect that appears in the joining area between the conical reduction and the cylindrical mantle, as a result of the different thicknesses of the two metal sheets (see figure 9 and 10).

A key role in determining with great accuracy of the tensions had the possibility, offered by ANSYS, to model a nonlinear and temperature dependent behavior of the sheet metal behavior. The performed analysis revealed the necessity of an interdisciplinary approach, in order to assure an accurate model and reliable results.

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